# CROSS-TALK IN QPSK COMMUNICATION SYSTEMS[1]

Tien Manh Nguyen and Yvette Owens
National **Aeronautics and Space Administration**
**Jet Propulsion Laboratory**
**California Institute of Technology**
**4800** Oak Grove Drive
Pasadena, CA 91109

## ABSTRACT

This report investigates the effects of cross-talk on the Bit Error Rate (BER) performance of QPSK communication systems, There are four different sources that can cause cross-talk in QPSK systems, namely, a band-limited channel, asymmetry in filters, phase imbalance between the channels, and imperfect carrier tracking. This report emphasizes the last two problems (where either phase imbalance in the local VCOS or imperfect carrier tracking exists). The BER is derived as a function of phase imbalance (or phase error caused by imperfect carrier tracking) for both QPSK and Unbalanced QPSK (UQPSK). However, numerical results for QPSK only are presented. Numerical results are presented by a set of curves that can be used to: ( 1 ) write a specification for an acceptable phase, imbalance between the In-phase (I) and Quadrature (Q) channels; and (2) specify the maximum allowable phase jitter produced by the carrier tracking loop.

# 1. Introduction

The topic on cross-talk in QPSK communications systems has been treated by various authors [1-4]. Cross-talk can cause serious performance degradation in QPSK communications systems, There are four different sources that cause cross-talk to occur in QPSK systems. They are:

(1) Bandlimitted Channel: It is well-known that unfiltered QPSK signals have constant-amplitude envelope versus time characteristics. In ideal conditions, e.g., ideal limiting amplifier and unlimiting bandwidth channel, the system performance is the same as if the prelimited filter and the limiting amplifier were absent. However, as the prelimiterbandwidth becomes narrower to the stage where significant signal energy is filtered away, the I and Q waveforms become seriously distorted and the signals acquire Amplitude Modulation (AM). The limiting amplifier removes the AM acquired during filtering, and in doing so further distorts the I and Q waveforms. As a result of this process, cross-talk between the waveforms occurs [1].

(2) Asymmetry in Filters: If the response of the filters (preliming filter or low pass filter, etc) are not symmetric about the center frequencies, a pure cosine (or sine) input signal will produce both sine and cosine terms at the output. As a result of this asymmetry, cross-talk between the I and Q channels has occurred [2].

(3) Phase Imbalanced Between the Channels: when the phase between the I and Q channels is not the same, the signal in the I channel can leak into the Q channel or vice versa. This process causes cross-talk between the channels.

(3) Imperfect Carrier Tracking: for QPSK systems, the imperfect carrier tracking can cause degradation from two sources. Firstly, it can cause degradation in the desired data signal by $\cos[F(t)]$ (where F(t) denotes the carrier tracking phase jitter) and secondly, it can cause crosstalk so that both the desired data and the opposite data channel appear in the desired data matched filter (where it can add or substract to the filter output) [3-4].

This paper investigates the last two cases, i.e., phase imbalance between the local VCOS and imperfect carrier tracking, Probability of error for both QPSK and QPSK will be derived. However, numerical results are presented only for QPSK for future recommendations to the CCSDS.

## 2. Cross-talk Due to the Phase Imbalanced between the Channels

The phase imbalance between the channels occurs When the phase shifter at the receiver is no longer operated in linear region due to aging or heating (see Figure 1). Due to the phase imbalance between the channels, the signal in the I-channel leaks into the Q-channel causing potential performance degradation in that channel. In this section we will consider the case when the signal in the Q-channel leaks into the I-channel, and that the data rate on the I-channel is n times the data rate on the Q-channel.

Let's assume that the received signal plus noise is modeled as

$$x(f) = \sqrt{2P_I}\,d_I(t)\sin(\omega_C t) + \sqrt{2P_Q}\,d_Q(t)\cos(\omega_C t) + n(t) \tag{1}$$

where n(t) is the AGWN with one-sided power spectral density $N_0$, $P_I$ and $P_Q$ are the I and Q channel power, $d_I(t)$ and $d_Q(t)$ are data sequences with symbol duration of $T_I$ and $T_Q$ sec, respectively.
To simplify the analysis as well as the hardware for practical implementation, we will consider the case that

$$\frac{T_I}{T_Q} = \frac{R_Q}{R_I} \quad n, \; n \; an \; interger \tag{2}$$

where $R_Q$ and RI are respective data rate of each channel. If we assume that the signal in the Q-channel leaks into the I-channel and no signal in the I-channel gets into the Q-channel. Let f be the phase imbalance between the I and Q channels. Using References 3 and 4, the conditional bit error probability in the I-channel can be shown to have the following form

$$P(E_I/\phi) = \frac{1}{2^n}\sum_{i=0}^{n} \binom{n}{i} Q\left[ \sqrt{\frac{2E_{bI}}{N_0}}\cos(\phi) - \left(\frac{n-2i}{n}\right)\sqrt{\frac{2nE_{bQ}}{N_0}}\sin(\phi) \right] \tag{3}$$

where

$$\frac{E_{bI}}{N_0} = \frac{P_I T_I}{N_0} \quad Bit \; SNR \; in \; I- \; Channel \tag{4}$$

$$\frac{E_{bQ}}{No} = \frac{P_Q T_Q}{N_0} \quad Iii/ \; SNR \; in \; Q\text{-}Chann$$

Note that Eqn (3) was derived based on the assumption that the data formats for both channels are NRZ and that the symbol synchronization is perfect, Furthermore, the binomial coefficient appeared in Eqn (3) because there are n symbols of $T_Q$ during $T_I$ seconds, and there are i negative symbols with n-i positive symbols so that the matched filter output of the cross-product (did) is equal to (n-2i)/n. This value occurs with a probability

The conditional bit error probability in the Q channel can be shown to be

$$P(E_Q/\phi) = Q\left[\sqrt{\frac{2E_{bQ}}{N_0}}\right] \tag{6}$$

If we assume that the signal in the I-channel leaks into the Q-channel and no signal in the Q-channel gets into the I-channel, then we can show that the conditional bit error probability in the Q-channel is given by

$$P(E_Q/\phi) = \frac{1}{2}\left(Q\left[\sqrt{\frac{2E_{bQ}}{N_0}}\cos(\phi) - \sqrt{\frac{2nE_{bQ}}{N_0}}\sin(\phi)\right] + Q\left[\sqrt{\frac{2E_{bQ}}{N_0}}\cos(\phi) + \sqrt{\frac{2nE_{bQ}}{N_0}}\sin(\phi)\right]\right) \tag{7}$$

The conditional bit error probability in the I-channel becomes

$$P(E_I/\phi) = Q\left[\sqrt{\frac{2E_{bI}}{N_0}}\right] \tag{8}$$

Note that the Q(.) function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}\,dt \tag{9}$$

The relationship between the Q(.) funtion and the erfc(.) is given as

$$Q(x) = \frac{1}{2}erfc\left(\frac{x}{\sqrt{2}}\right) \tag{lo}$$

For balanced power among the channels and equal data rate, i.e., QPSK (as contrast to Unbalanced QPSK, UQPSK), the conditional bit error probability in the I-channel, assuming the signal in the Q-channel leaks into the I-channel and no signal in the I-channel gets into the Q-channel, becomes

$$P(E_I/\phi) = \frac{1}{2}\left(Q\left[\sqrt{\frac{2E_b}{N_0}}\cos(\phi) - \sqrt{\frac{2E_b}{N_0}}\sin(\phi)\right] + Q\left[\sqrt{\frac{2E_b}{N_0}}\cos(\phi) + \sqrt{\frac{2E_b}{N_0}}\sin(\phi)\right]\right) \tag{11}$$

Similarly, the conditional bit error probability for the other case can be derived by setting n = 1, $E_{bQ}/N_0 = E_{bI}/N_0 = El,/No$, and the result is identical to EQn ( I I ).

### 3. **Cross-talk Due to Imperfect Carrier Tracking**

This case has been investigated in detail in **[3-4].** AS mentioned earlier, imperfect carrier tracking can cause degradation from two sources, namely, the phase error degrades the desired data signal and the phase error also causes interchannel interference. If we assume perfect bit synchronization and NRZ data formats for both channels, the conditional probability of error in the I-channel for UQPSK is given by [3-4]

$$P(E_I/(\Phi(t) \equiv \phi)) = \frac{1}{2^n} \Sigma_{i-0}^n \binom{n}{i} Q\left[\sqrt{\frac{2E_{bI}}{N_0}} \cos(\phi) \cdot \frac{n-2i}{n} - \right) \sqrt{\frac{2nE_Q}{N_0}} \sin(\phi) \tag{12}$$

and the conditional probability of bit error in the Q-channel for UQPSK is found to be [3-4]

$$P(E_Q/\Phi(t) \equiv \phi) = \frac{1}{2}\left( Q\left[\sqrt{\frac{2E_{bQ}}{N_0}} \cos(\phi) - \sqrt{\frac{2nE_{bQ}}{N_0}} \sin(\phi)\right] + Q\left[\sqrt{\frac{2E_{bQ}}{N_0}} \cos(\phi) + \sqrt{\frac{2nE_{bQ}}{N_0}} \sin(\phi)\right]\right) \tag{13}$$

For QPSK, the conditional bit error probability for I-channel becomes identical to the Q-channel and it is given by

$$P(E/\Phi(t) \equiv \phi) = \frac{1}{2}\frac{1}{2\pi}Q\left[\sqrt{\frac{2E_b}{N_0}} \cos(\phi) - \sqrt{\frac{2E_b}{N_0}} \sin(\phi)\right] + Q\left[\sqrt{\frac{2E_b}{N_0}} \cos(\phi) + \sqrt{\frac{2E_b}{N_0}} \sin(\phi)\right]\right) \tag{14}$$

### 4. **Numerical Results**

Due to common interest in the CCSDS community, numerical results for QPSK are presented in this section. For this particular case the conditional bit error probability for phase imbalance is identical for imperfect carrier tracking. However, for phase imbalance case, only one channel is affected by the phase imbalance depending on which channel is off as compared to the other channel. Since we are concerned with bit error rate degradation, hence Eqn (14) is important in the investigation of the effect of crosstalk in QPSK systems. Plot of Eqn **(14) is** shown in Figure 2. This figure plot the Bit Error Rate (BER) as a function of bit SNR with the phase imbalance (or phase jitter) as a parameter. The numerical results show that as the phase imbalance (or phase jitter) increases, the BER performance degradation also increases. Table 1

summarizes the results for various values of phase imbalance and BERs.

## 5. Conclusions and Recommendations

Based on the numerical results presented in Figure 1 and Table 1, it is clear that when the phase imbalance or phase jitter greater than 2 degrees, the bit SNR degradation is greater than or equal to 0.1 dB for BER <10-3. Therefore, in order to keep the bit SNR degradation due to the phase imbalance less than 0.1 dB, the phase imbalance between the I and Q channels should be kept less than or equal to 2 degrees.

**Table 1: Bit SNR Degradation for Various Values of Phase Imbalanced or Phase Jitter**

| Phase Imbalance (Degree) | Bit | SNR | Degradation |
|---|---|---|---|
| | $13 ER=10^{-3}$ | $BER=10^{-4}$ | $BER=10^{-5}$ |
| 2 | 0.10 dB | 0.10 dB | 0,11 dB |
| 4 | 0.25 dB | 0.30 dB | 0.35 dB |
| 6 | 0.50 dB | 0.62 dB | 0.70 dB |
| 8 | 0.85 dB | 0.95 dB | 1.07 dB |

### References

[1]        K. Feher, Digital Communications. Satellite/Earth Station Engineering, Prentice-Hall, Englewood Cliffs, New Jersey, 1983

[2]        L. B. Milstein, Digital Communications Systems, unpublished note, University of California San Diego, La Jolla, CA, 1981.

[3]        Osborne, H. C., "Effects of Noisy Reference on Coherent Detection of Unbalanced Signals", Proceedings NTC 1978, Birmingham, Ala., December 3-5, 1978.

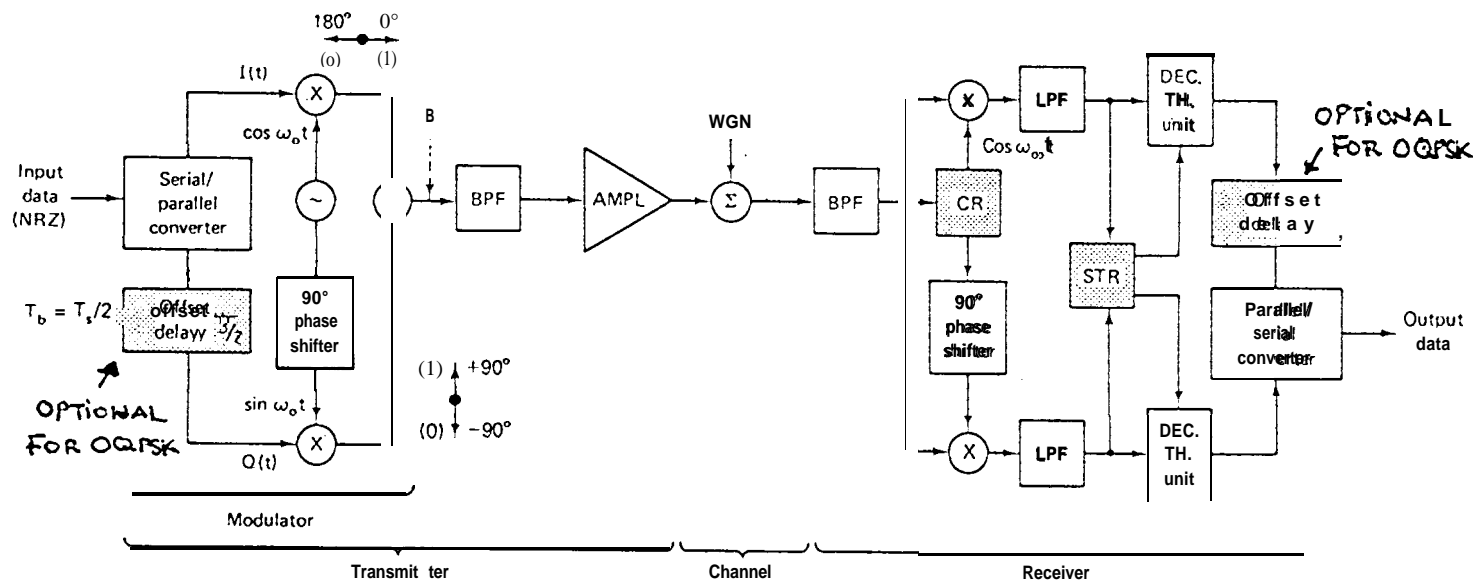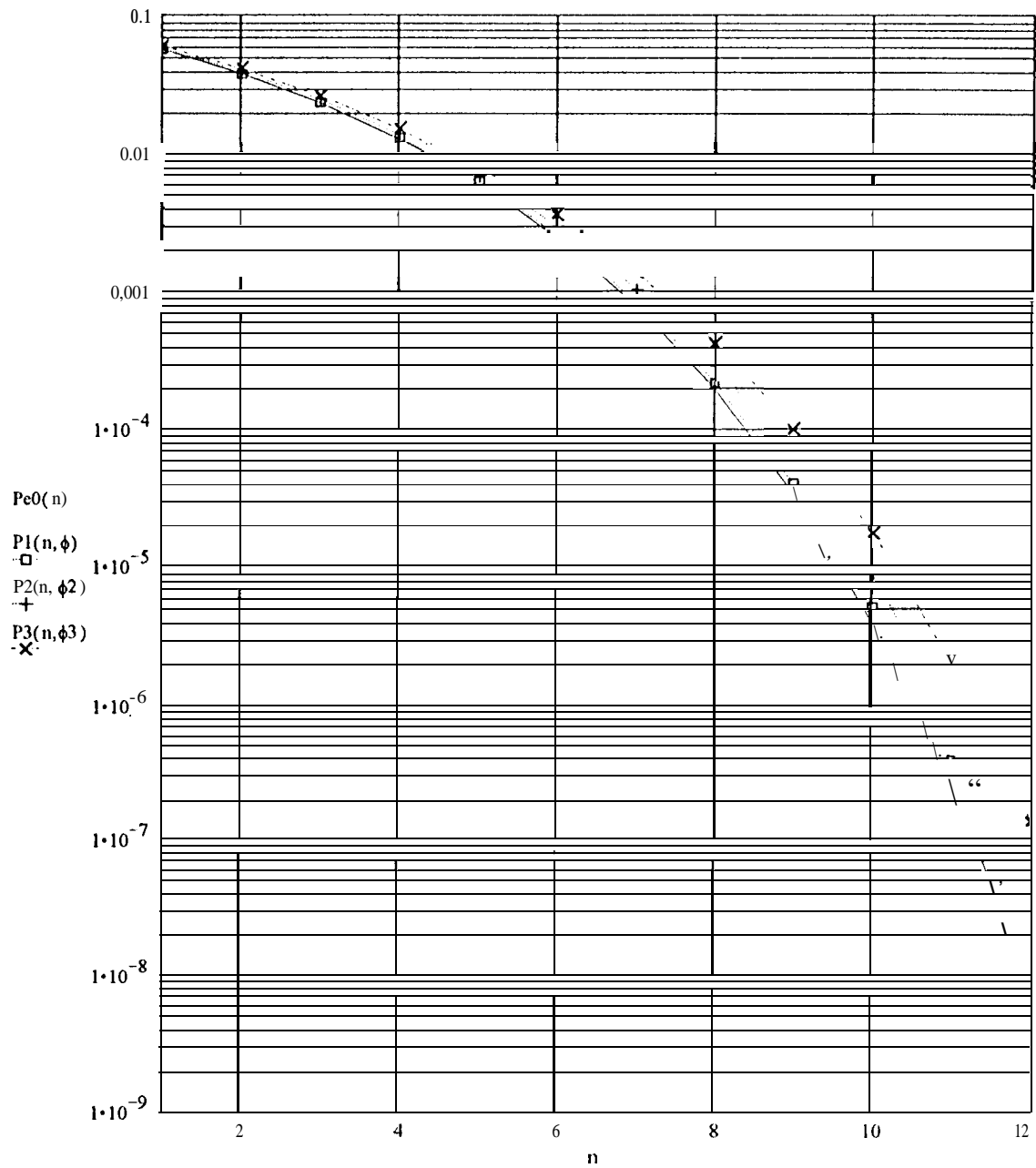[4]        J. K. Holmes, Coherent Communications Systems, John Wiley & Son, New York, 1982.

**Figure 1.** Typical **QPSK** Communications Modem-The Delay is
Optional for Offset QPSK Systems

LEGEND:

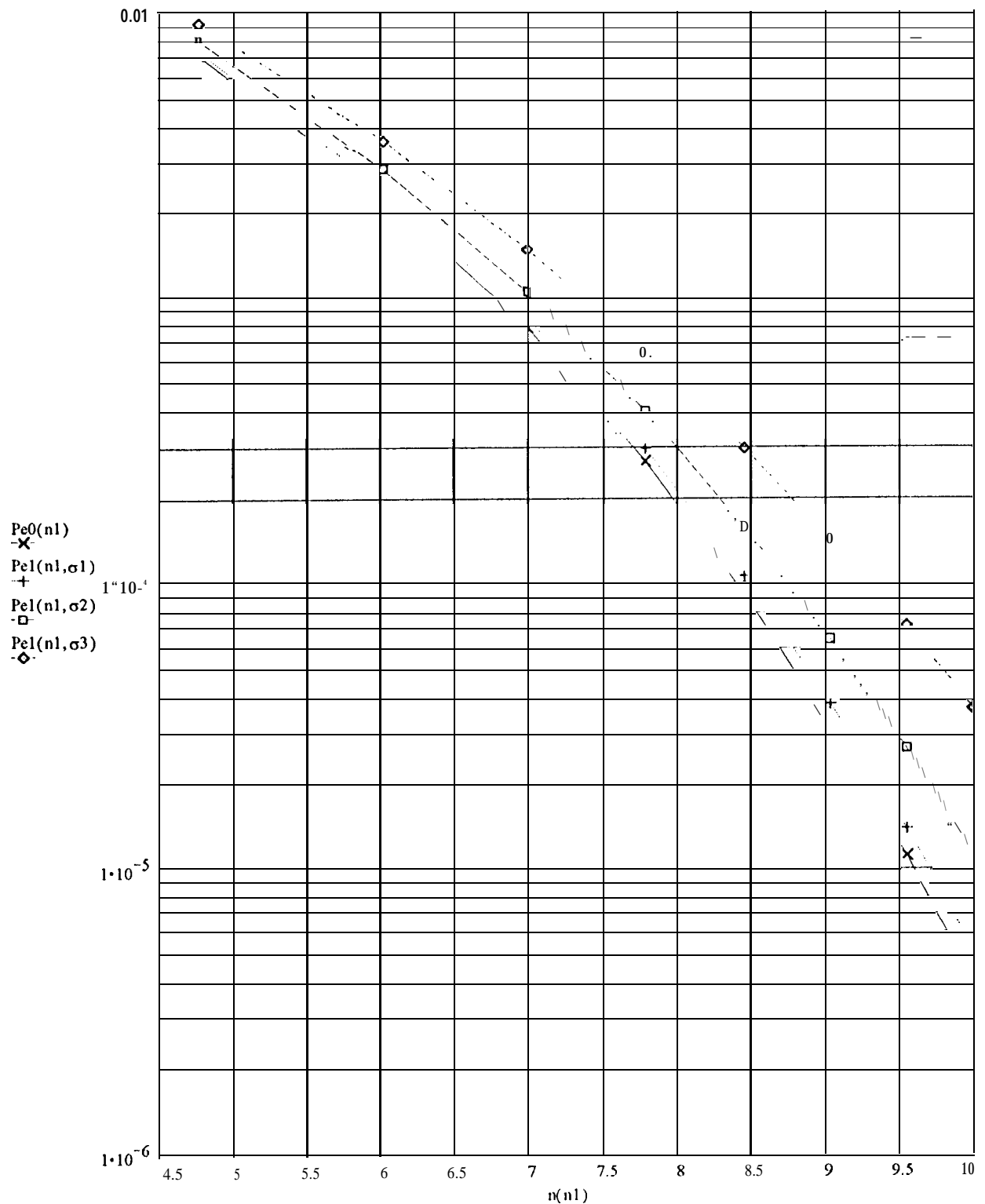$Pe0(n) = $ BER for Ideal  Case

$P1(n, \phi1) = $ BER at 2 Deg Phase  imbalanced;

$P2(n, \phi2.) = $ BER at 4 Deg Phase imbalanced;

$P3(n, \phi3) = $ BER at 6 Deg Phase imbalanced

$n = $ Bit SNR in dB

92

**Figure 3. Average Bit Error Rate Probability vs Eb/No for Various Values of RMS Phase Jitter**



**LEGEND:**

**Pe0(n1) = BER for Ideal Case**

$Pe1(n1,\sigma1)$ = **Average BER at 2 Deg RMS Phase Jitter**

$Pe1(n1,\sigma2)$ = **Average BER at 4 Deg RMS Phase Jitter**

$Pe1(n1,\sigma3)$ = **Average BER at 6 Deg RMS Phase Jitter**

**n(n1) = Bit SNR in dB**

93